Switch of design variables: a cost-effective identification of the Pareto front in inverse magnetics

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A new optimization method, combining design of experiments with evolutionary computing, is proposed: it handles a set of design variables, the size of which changes during the process: initially, most sensitive variables are activated; subsequently, the whole set of variables is activated. The optimal synthesis of a magnetic field for magneto-fluid treatment is considered as the case study.

***Index Terms*— Evolutionary computing, multiobjective optimal design, sensitivity analysis, finite elements, magnetic field synthesis.**

I. Introduction

When tackling problems of optimal shape design in magnetics, characterized by Finite Elements (FE) field analyses for solving the associated direct problem, the so-called parametric approach is normally adopted: a set of many design variables, defining the unknown shape of the device to synthesize, is searched for, usually exploiting algorithms of evolutionary computing. In particular, in multi-objective problems, the search for the Pareto-optimal front of the problem is based on popular algorithms like *e.g.* Non-Dominated Genetic Algorithm (NSGA) or Multi Objective Particle Swarm Optimization (MOPSO) [1]–[5]. When the dimensionality of the design problem, mainly dictated by the number of design variables, is high, a combinatorial increase of feasible design points occurs: in case, cost-effective procedures of optimization can be implemented exploiting *e.g.* surrogate models, *i.e.* identifying response surfaces that replace the objective functions at a lower cost [7]–[8]. Alternatively, one might think of subdividing the design variable set in *e.g.* two subsets, in such a way that the first part of the optimization is driven by the most sensitive variables, in order to approach fast the region of Pareto-optimal solutions, and then switching to the full set of variables, in order to focus on the details of the search region. The principle of progressively enhancing the design variable set emulates what happens in the real-life operation of a device designer; moreover, it appears to be suited to evolutionary computing, when substantial variations of the objectives take place in the first group of iterations. An algorithm implementing the switched variable approach is here proposed.

II. Proposed Optimization Method

The proposed optimization method combines Design Of Experiments (DOE) [9] with NSGA [2], and acts on a set of design variables the size of which changes during the optimization process. The SV-NSGA-DOE (SV, switched variables) algorithm works as shown in Fig 1. At first the DOE analysis is used to evaluate what are the design variables that are more sensible within the prescribed bounds. Exploiting Placket-Burmann tables [9], a cost-effective evaluation of sensitivity is performed: a number N_{DOE} of FE analyses makes it possible to approximate the sensitivity S_{xi} of each out of N_V design variables. Then, the average sensitivity, S_m, is:

$$S_m = \frac{1}{N_V}\sum_{i=1}^{N_V} S_{xi} \tag{1}$$

The most sensitive design variables are defined as those for which $S_{xi} > S_m$, $i=1,..N_V$. This way, a reduced set of design variables is activated. The sensitivity evaluation takes place only once, before the NSGA-based optimization – acting on the reduced set of design variables - is started. After a number of iterations, it is decided to switch, and the full set of design variables is eventually activated.

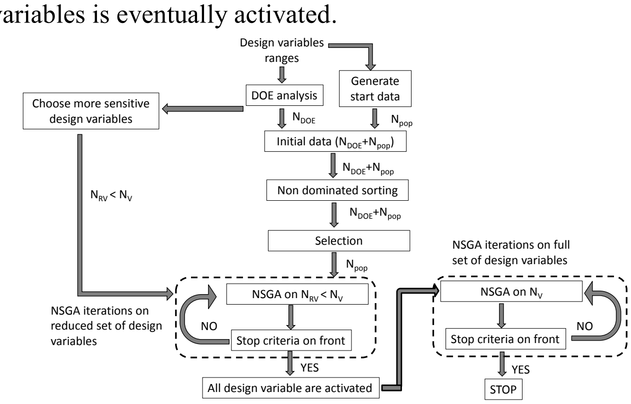

Fig. 1. Flow chart of SV-NSGA-DOE algorithm.

Therefore, NSGA optimization is performed in two steps (Fig. 1): in the former only the reduced set of most sensitive design variables is considered, while in the latter the full set of design variables is considered. N_{pop} individuals are selected in the initial population, which N_{DOE} individuals used for the DOE analysis are added to. Therefore, the initial population contains N_{DOE} + N_{pop} individuals. A first selection, ruled by non-dominated sorting, reduces the population size to N_{pop}. At each iteration the stopping criterion is evaluated: if it is fulfilled, the algorithm 'switches' and all the design variables are activated in the optimization procedure; the population size remains equal to N_{pop} and the ranges of design variables already active before switching do not change as well. The optimization ends when the stopping criterion is again fulfilled.

The NSGA algorithm generally stops when the maximum number of iteration is achieved. In this work, an automatic stopping criterion, based on the evaluation of the front displacement, is implemented. In particular, in the chromosome of the *h-th* iteration the distance of each individual from the origin of the objective space (utopia point), $d_j(h)$ $j=1,\ldots,N_{pop}$, is computed; then, the average distance, $d_m(h)$, is evaluated as:

$$d_m(h) = \frac{1}{N_{pop}} \sum_{j=1}^{N_{pop}} d_j\,(h) \qquad (2)$$

This average distance is compared with the one at the previous generation, $d_m(h\text{-}1)$. If the relative difference is lower than a prescribed threshold, $d_{\%,th}$ (*e.g.* 1%), the front is considered to be stationary and the NSGA can either 'switch' to the full set of design variables (first step) or stop (second step). In practice, the switch or stop event is decided by averaging *M* percentage difference, $d_{\%}(h)$.

III. Case Study: Power Inductor for Magnetic Nanoparticle Heating

Fig. 2 shows the cross section of the axi-symmetric geometry of the device. The Petri dish is placed in a thermally insulated box where a water flow keeps the temperature of the system at 37°C. The magnetic field device is a two-turn inductor with five ferrite blocks placed as in Fig. 2 in order to concentrate the magnetic flux lines. The magnetic field analysis problem, based on the **A**-V formulation, is solved in time-harmonics conditions using a FE code [10]-[11]. The inductor is supposed to be supplied by means of a voltage of 600 V. A typical FE mesh exhibits 23,000 nodes and 13,000 elements.

The aim of the optimization problem is to minimize both (f_1) the inhomogeneity of the magnetic field, *H*, in the Petri dish as in [12] with a tolerance interval of ±10 A/m, and (f_2) the inverse of the average magnetic field strength in the Petri.

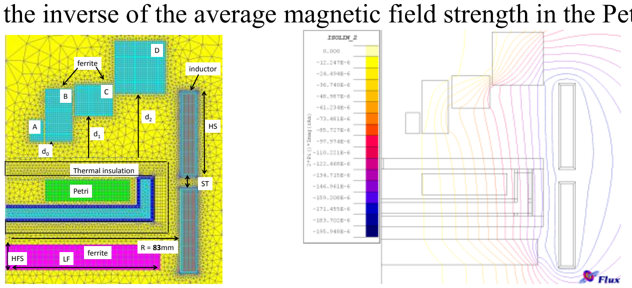

Fig. 2. Model geometry, mesh detail, design variables, and magnetic flux lines

IV. Results

Fig. 3 shows the results obtained starting from the same initial population of N_{pop} individuals and applying:

a. NSGA-II algorithm (results referred as #_NSGA);

b. SV-NSGA-DOE (results referred as #_SV);

c. NSGA-II algorithm incorporating the DOE-evaluated individuals in the initial population (named NSGA-DOE with, results referred as #_DOE).

In the presented example only three design variables are selected as more sensible in the reduced set (d_2, ST and HFS in Fig 2). In Fig. 3 it appears that the Pareto front obtained using the proposed SV-NSGA-DOE algorithm is broader than the one fond via a standard NSGA-II algorithm. Also the incorporation of the extra individuals used in the initial DOE analysis contributes to enhance results. Moreover, it can be noted that the solutions obtained by means of reduced design variable set are located at the ends of the Pareto front.

In Fig. 4 the evolution of the f_1 objective function is shown, considering at each iteration the best out of N_{pop} values. It can be noted that substantial function variations take place both before and after the switch of design variables.

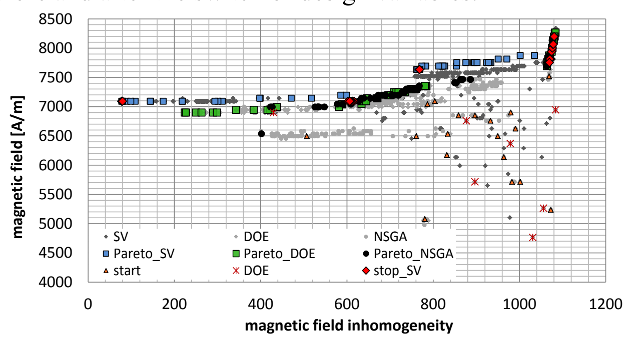

Fig. 3. Pareto fronts obtained starting from the same initial population (start) and applying methods a, b and c. SV, DOE and NSGA generated individuals, Pareto_# individuals on Pareto front, DOE individuals of the DOE analysis, stop_SV individuals at switching iteration using b. NSGA stops after 30 iterations, NSGA-DOE after 48, and SV-NSGA-DOE after 53 (switch at 37).

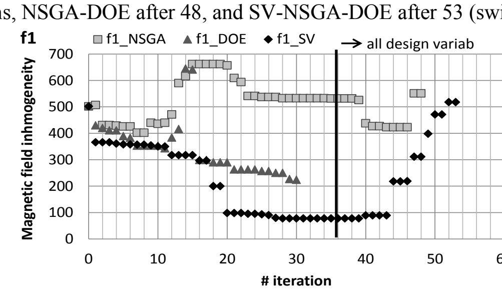

Fig. 4. History of the f_1 objecitve function: the switch line from reduced to full set of design variables is shown.

V. References

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